Table 5.15. A 2^3 Design in Blocks of Size 2, a Good Arrangement

Run Number	Experimental Variable 1	2	3	Block Variable 4 = 12	5 = 13	Experiment Arranged in Four Blocks: Block	1	2	3	Run
1	−	−	−	+	+	I	+	−	−	2
2	+	−	−	−	−		−	+	+	7
3	−	+	−	−	+					
4	+	+	−	+	−	II	−	+	−	3
5	−	−	+	+	−		+	−	+	6
6	+	−	+	−	+					
7	−	+	+	−	−	III	+	+	−	4
8	+	+	+	+	+		−	−	+	5
						IV	−	−	−	1
							+	+	+	8

5.17. LEARNING BY DOING

> One must learn by doing the thing; for though you think you know it, you have no certainty until you try.
>
> SOPHOCLES

You may not be currently involved in a formal project to which you can apply experimental design. In that case we believe you will find it rewarding at this point to plan and perform a home-made factorial experiment and collect and analyze the data. We have regularly assigned such a project to our classes when teaching this material. The students have enjoyed the experience and have learned a great deal from it. We have always left it to the individual to decide what he or she wants to study.

What follows paraphrases a report by a student; Norman Miller, of the University of Wisconsin.

Description of Design

I ride a bicycle a great deal and thought that it might be interesting to see how some of the various parts of the bike affect its performance. In particular, I was interested in the effects of varying the seat height, varying the tire pressure, and using or not using the generator.

In deciding on a design, a 2^3 factorial design immediately came to mind. I expected a large amount of variability, so I decided to replicate the eight data points in order to be able to estimate the variance. Since I wished to do the experiment in a reasonable length of time, I decided to make four runs a day. Although a long run might be desirable in that various types of terrain could be covered, I felt that four long runs a day would be out of the question. I did not have time to make them, I feared that I would tire as the day progressed, making the runs late in the day incompatible with the earlier runs, and I also feared that with the increased exercise my own strength and endurance would increase as the experiment progressed. As a result of the above considerations I decided to do a rather short one and a half block uphill run. My test hill was on Summit Avenue in Madison, starting at Breese Terrace. This hill is of such an inclination that it all must be taken in first gear, thus eliminating gear changes as a possible source of experimental error. I tried to follow a schedule of 9:00 A.M., 11:30 A.M., 2:00 P.M., and 4:30 P.M. As can be seen from the data sheet (Table 5.16), I deviated from the schedule quite a bit; however, I did avoid making runs right after meals. Note that on day 1 I was able to make only three runs because of bad weather. I made up the lost run on day 3. I assigned the variable levels as shown on the data sheet. The seat height of 30 inches is the maximum elevation

Table 5.16. Data from Bicycle Experiment

Setup	Seat Height	Generator	Tire Pressure	Pulse Before	Pulse After	Time (sec)	Day	Hour	Notes
1	−	−	−	76	135	51	3	11:40 A.M.	36°F
2	−	−	−	76	133	54	4	9:40 A.M.	37°F
3	+	−	−	76	132	41	2	12:35 A.M.	23°F, snow on road
4	+	−	−	79	137	43	2	4:05 P.M.	25°F, snow melting
5	−	+	−	77	134	54	3	2:40 P.M.	36°F
6	−	+	−	77	133	60	2	9:50 A.M.	fourth run, 23°F, snow
7	+	+	−	76	130	44	3	9:10 A.M.	26°F, road clear
8	+	+	−	79	139	43	4	3:35 P.M.	40°F
9	−	−	+	76	105	50	1	9:15 A.M.	first run
10	−	−	+	80	144	48	4	6:05 P.M.	40°F
11	+	−	+	77	139	39	3	6:35 P.M.	32°F
12	+	−	+	78	139	39	4	11:40 A.M.	41°F
13	−	+	+	78	137	53	3	4:50 P.M.	35°F, rain
14	−	+	+	79	125	51	1	4:25 P.M.	third run
15	+	+	+	80	122	41	1	11:40 A.M.	second run
16	+	+	+	77	133	44	2	6:35 P.M.	21°F

	Seat height (inches from center of crank)	Generator	Tire pressure (psi)
−	26	off	40
+	30 (maximum height)	on	55

of the seat on my bike. I could ride with it higher, so my bike is in effect too small for me.

I measured two responses. First, I measured the speed in seconds to climb the hill. I naturally was trying to ride as fast as possible. Second, I hoped to get some idea of the amount of energy I was expending during the run by measuring my pulse right before and right after the run. I always rode around two blocks at a slow pace before each run to get warmed up. I then stopped at the bottom of the hill and let my pulse settle to the range of 75 to 80 per minute. I would then make my run, timing it on a watch with a large sweep second hand. At the top of the hill, I took my pulse again. I randomized the experiment by writing the numbers 1 through 16 on pieces of cardboard, putting them in a small box, shaking up the box, and drawing out one number before each experiment to determine the setup of the variables.

Two possible criticisms of the experiment came to mind. First, since I knew the setup of the variables, I might bias the experiment to "come out right." I could not think of a way around this. It would have been quite an imposition to ask a friend to set up the bicycle 16 times! In addition, even if a friend had changed the variables each time, I could have heard the generator operating (or seen that the light was burning) and could have seen or felt the height of the seat. Another rider would probably be just as biased as I would be. In addition, who would ride up a hill 16 times for me? So I just decided to try to be as unbiased as possible. The second criticism involves the rather large temperature range encountered during the experiment. Naturally oil tends to be more viscous at low temperatures. I considered that this problem was probably less acute than it might have been, however, since I used silicone oil to lubricate the bike.

The original data sheet appears in Table 5.16. Notice that there is a problem with the data on pulse. Setup 9, the first run, shows my pulse at the top of the hill to be 105 per minute. Setup 15, the second run, has it at 122 per minute. Setup 14, the third run, has it at 125 per minute, and setup 6, the fourth run, at 133 per minute. What was happening was the following. When I arrived at the top of the hill, my heart was beating quite fast, but it started slowing down quickly as soon as I stopped. With practice, I was simply starting to measure my pulse sooner. Since the pulse data are damaged by this obvious error, I have chosen to ignore them in the following analysis. The readings taken at the bottom of the hill did prove valuable since they served as a gauge of when my body was in a "normal" state.

Analysis of Data

First, I plotted the average times on a cube (Fig. 5.17) and looked at them. Then I calculated:

1. Estimated variance of each of 16 observations = 4.19 square seconds with eight degrees of freedom
2. Main effects and interactions

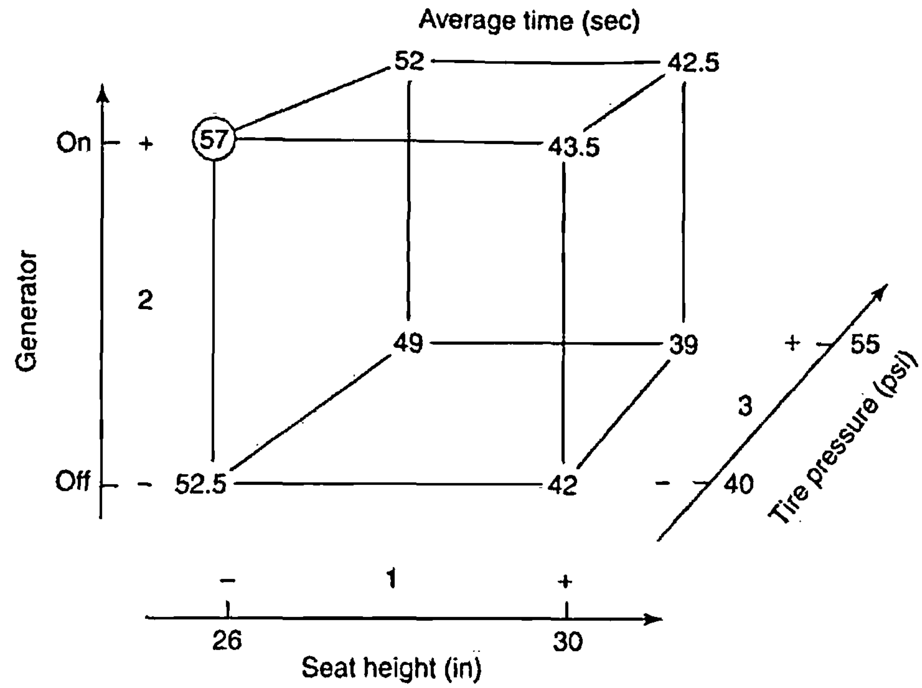

Figure 5.17. Results from bicycle experiment. Response is average time to ride bicycle over set course.

3. Estimated variance of effect = 4.19/4 = 1.05
4. Individual 95% confidence intervals for true values of effects*

Effects		
1 (seat)	=	-10.9 ± 2.4
2 (generator)	=	3.1 ± 2.4
3 (pressure)	=	-3.1 ± 2.4
12 (seat × generator)	=	-0.6 ± 2.4
13 (seat × pressure)	=	1.1 ± 2.4
23 (generator × pressure)	=	0.1 ± 2.4
123 (seat × generator × pressure)	=	0.9 ± 2.4

I believe that the value of 57 circled on the cube is probably too high. Notice that it is the result of setup 6, which had a time of 60 seconds. Its comparison run (setup 5) was only 54 seconds. This is the largest amount of variation in the whole table, by far. I suspect that the correct reading for setup 6 was 55 seconds, that is, I glanced at my watch and thought that it said 60 instead of 55 seconds. Since I am not sure, however, I have not changed it for the analysis. The conclusions would be the same in either case.

Obviously a good stopwatch would have helped the experiment, but I believe that the results are rather good considering the adverse weather and poor timer.

*Notice these provide 95% confidence limits. They are *not* standard errors.

A slightly better design might include a downhill and a level stretch to provide more representative terrain. Probably only two runs could be safely made each day in such a case.

Conclusions

Raising the seat to the high position cuts the time to climb the hill by about 10 seconds. Turning on the generator increases the time by about one-third of that amount. Inflating the tires to 55 psi reduces the time by about the same amount that the generator increases it. The variables appear to be substantially free of interactions.

Since I could ride with the seat at an even higher position, I am planning to modify the bike to achieve this. I also plan to inflate my tires to 60 psi, which is the maximum for which they are designed.

Exercise 5.13. In the bicycle experiment

(a) Identify the two runs that were accidentally switched

(b) Suppose the design had been carried out as intended, if the four days on which the data was obtained are regarded as blocks, would this have been an *orthogonally* blocked experiment?

(c) Make an analysis of variance and show by how much the residual sum of squares would be reduced by blocking.

5.18. SUMMARY

The examples show the value of even very simple factorial experiments. Conclusions from such designs are often best comprehended when the results are displayed geometrically. These designs used in combination with geometric visualization are valuable building blocks in the natural sequential process of learning by experiment. They allow the structure of our thinking to be appropriately modified as investigation proceeds.

APPENDIX 5A. BLOCKING LARGER FACTORIAL DESIGNS

The 2^3 factorial design was used to illustrate the principle of blocking designs by means of confounding. These principles, however, are equally applicable to larger examples. Table 5A.1 provides a list of useful arrangements. To understand how this table is used, consider a 2^6 factorial design containing 64 runs. Suppose that the experimenter wishes to arrange the experiment in eight blocks of eight runs each. The eight blocks, for example, might be associated with eight periods of time or eight batches of raw material.

Then the table suggests you use the arrangement

$$\mathbf{B}_1 = \mathbf{135}, \quad \mathbf{B}_2 = \mathbf{1256}, \quad \mathbf{B}_3 = \mathbf{1234}$$