

## STA305/1004 - Class 9

October 10, 2019

## Today's Class

- ▶ Design of an observational study versus experiment
- ▶ Maimonides' rule (an example of a natural experiment)
- ▶ Introduction to the propensity score

## The design phase of an observational study

Good observational studies are designed.

*An observational study should be conceptualized as a broken randomized experiment ... in an observational study we view the observed data as having arisen from a hypothetical complex randomized experiment with a lost rule for the [assignment mechanism], whose values we will try to reconstruct. Rubin (2007)*

## The design phase of an observational study

*Of critical importance, in randomized experiments the design phase takes place prior to seeing any outcome data. And this critical feature of randomized experiments can be duplicated in observational studies, for example, using propensity score methods, and we should objectively approximate, or attempt to replicate, a randomized experiment when designing an observational study. Rubin (2007)*

## Looking for Causal Treatment Effects

- ▶ Compare survival in patients who are treated with a drug therapy versus surgery in a randomized control trial.
- ▶ If patients that undergo surgery live 8 years and patients that take drug therapy live 5 years then the treatment effect is  $8-5=3$ .

## Observational versus Randomized Studies

- ▶ In observational studies the researcher does not randomly allocate the treatments.
- ▶ Randomization ensures subjects receiving different treatments are comparable.

## Observational Studies

- ▶ Suppose that we have an outcome measured on two groups of subjects (treated and control).
- ▶ We want to make a fair comparison between the treated group and the control group in terms of the outcome.
- ▶ We can obtain covariates that describe the subjects before they received treatments, but we can't ensure that the groups will be comparable in terms of the covariates.

## Importance of Randomization

- ▶ Randomization tends to produce relatively comparable or “balanced” treatment groups in large experiments.
- ▶ The covariates aren't used in assigning treatments in an experiment.
- ▶ There's no deliberate balancing of the covariates – it's just a nice feature of randomization.
- ▶ We have some reason to hope and expect that other (unmeasured) variables will be balanced, as well.



## Smoking cessation and weight gain

What is the effect of smoking on weight gain?

- ▶ Data was used from The National Health and Nutrition Examination Survey Data I Epidemiological Follow-up Study (NHESFS) survey to assess this question.
- ▶ The NHESFS has information on sex, race, weight, height, education, alcohol use, and intensity of smoking at baseline (1971-75) and follow-up (1982) visits.
- ▶ The survey was designed to investigate the relationships between clinical, nutritional, and behavioral factors.
- ▶ A cohort of persons 25-74 who completed a medical exam in 1971-75 followed by a series of follow-up studies.
- ▶ Would it be possible to conduct a randomized study to evaluate the treatment effect?

## Smoking cessation and weight gain

	Cessation (A=1)	No cessation (A=0)
age, years	46.2	42.8
men, %	54.6	46.6
white, %	91.1	85.4
university, %	15.4	9.9
weight, kg	72.4	70.3
Cigarettes/day	18.6	21.2
year smoking	26.0	24.1
little/no exercise, %	40.7	37.9
inactive daily life, %	11.2	8.9

## Maimonides' Rule



- ▶ Angrist and Lavy (1999) published an unusual study of the effects of class size on academic achievement.
- ▶ Causal effects of class size on pupil achievement is difficult to measure.
- ▶ The twelfth century Rabbinic scholar Maimonides interpreted the the Talmud's discussion of class size as:

*“Twenty-five children may be put in charge of one teacher. If the number in the class exceeds twenty-five but is not more than forty, he should have an assistant to help with instruction. If there are more than forty, two teachers must be appointed.”*

# Maimonides' Rule



- ▶ Since 1969 it has been used to determine the division of enrolment cohorts into classes in Israeli public schools.
- ▶ Class size is usually determined by affluence or poverty of a community, enthusiasm or skepticism about the value of education, special needs of students, etc.
- ▶ If adherence to Maimonides's rule were perfectly rigid then with a single class size of 40 what would separate the same school with two classes whose average size is 20.5 is one student.

## Maimonides' Rule



- ▶ Maimonides' rule has the largest impact on a school with about 40 students in a grade cohort.
- ▶ With cohorts of size 40, 80, and 120 students, the steps down in average class size required by Maimonides' rule when an additional student enrolls are shown in the table.

Original Cohort Size	40	80	120
Class size with one additional student	20.5	27	30.25

## Maimonides' Rule



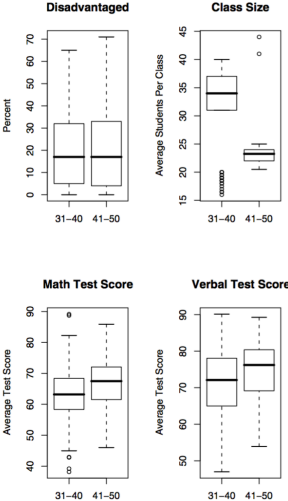
- ▶ Angrist and Lavy looked at schools with fifth grade cohorts in 1991 with between 31 and 50 students, where average class sizes might be cut in half by Maimonides' rule.
- ▶ There were 211 such schools, with 86 of these schools having between 31 and 40 students in fifth grade, and 125 schools having between 41 and 50 students in the fifth grade.

## Maimonides' Rule



- ▶ Among the 211 schools with between 31 and 50 students in fifth grade, the percentage disadvantaged has a slightly negative Kendall's correlation of  $-0.10$ .
- ▶ Strong negative correlations of  $-0.42$  and  $-0.55$ , respectively, with performance on verbal and mathematics test scores.
- ▶ For this reason, 86 matched pairs of two schools were formed, matching to minimize total absolute difference in percentage disadvantaged.

# Maimonides' Rule



**Fig. 1.1** Eighty-six pairs of two Israeli schools, one with between 31 and 40 students in the fifth grade, the other with between 41 and 50 students in the fifth grade, matched for percentage of students in the school classified as disadvantaged. The figure shows that the percentage of disadvantaged students is balanced, that imperfect adherence to Maimonides' rule has yielded substantially different average class sizes, and test scores were higher in the group of schools with predominantly smaller class sizes.



## Maimonides' Rule

86 pairs of Israeli schools that are matched as follows:

1. One school with between 31 and 40 students in the fifth grade ( $T = 0$ ),
2. One with between 41 and 50 students in the fifth grade ( $T = 1$ ), the pairs being matched for a covariate  $x =$  percentage of disadvantaged students.

## Maimonides' Rule

- ▶ Strict adherence to Maimonides' rule would require the slightly larger fifth grade cohorts to be taught in two small classes, and the slightly smaller fifth grade cohorts to be taught in one large class.
- ▶ Adherence to Maimonides' rule was imperfect but strict enough to produce a wide separation in typical class size.

## Maimonides' Rule

- ▶ In their study,  $(Y(1), Y(0))$  are average test scores in fifth grade if the fifth grade cohort was a little larger ( $T = 1$ ) or a little smaller ( $T = 0$ ).
- ▶ What separates a school with a slightly smaller fifth grade cohort ( $T = 0$ , 31–40 students) and a school with a slightly larger fifth grade cohort ( $T = 1$ , 41–50 students)?
- ▶ A handful of grade 5 students.

## Maimonides' Rule

- ▶ Seems plausible that whether or not a few more students enrol in the fifth grade is a haphazard event.
- ▶ An event not strongly related to the average test performance ( $Y(1)$ ,  $Y(0)$ ) that the fifth grade would exhibit with a larger or smaller cohort.
- ▶ Building a study in the way that Angrist and Lavy did, it seems plausible that the propensity of a larger/smaller class are fairly close in the 86 matched pairs of two schools.

## Maimonides' Rule

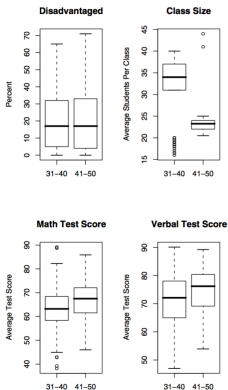
A 'natural experiment' is an attempt to find in the world some rare circumstance such that a consequential treatment was handed to some people and denied to others for no particularly good reason at all, that is, haphazardly.

## Maimonides' Rule

- ▶ It does seem reasonable to think that probability of a larger class are fairly close for the 86 paired Israeli schools.
- ▶ This might be implausible in some other context, say in a national survey in Canada where schools are funded by local governments, so that class size might be predicted by the wealth or poverty of the local community.

# Maimonides' Rule

The figure shows that treatment reflects the haphazard element, namely susceptibility to Maimonides' rule based on cohort size, rather than realized class size. Defining treatment in this way is analogous to an 'intention-to-treat' analysis in a randomized experiment.



**Fig. 1.1** Eighty-six pairs of two Israeli schools, one with between 31 and 40 students in the fifth grade, the other with between 41 and 50 students in the fifth grade, matched for percentage of students in the school classified as disadvantaged. The figure shows that the percentage of disadvantaged students is balanced, that imperfect adherence to Maimonides' rule has yielded substantially different average class sizes, and test scores were higher in the group of schools with predominantly smaller class sizes.

## The propensity score

- ▶ Covariates are pre-treatment variables and take the same value for each unit no matter which treatment is applied.
- ▶ For example, pre-treatment blood pressure or pre-test reading level are not influenced by a treatment that would alter blood pressure or reading level.

The propensity score is

$$e(\mathbf{x}) = P(T = 1|\mathbf{x}),$$

where  $\mathbf{x}$  are observed covariates.

The  $i^{th}$  propensity score is the probability that a unit receives treatment given all the information, recorded as covariates, that is observed before the treatment.



## The propensity score

- ▶ In experiments the propensity scores are known.
- ▶ In observational studies they can be estimated using models such as logistic regression where the outcome is the treatment indicator and the predictors are all the confounding covariates.

## The propensity score

Consider a completely randomized design with  $n = 2$  experimental units. One unit is assigned treatment one ( $T = 1$ ) and one unit is assigned to treatment two ( $T = 0$ ). The possible treatment assignments for the experimental units are:

$T_1$	$T_2$	$P(T_1)$	$P(T_2)$
0	0	0	0
0	1	0	0.5
1	0	0.5	0
1	1	0	0

Each unit's propensity score is ?.

## The propensity score

Consider a completely randomized design with  $n = 8$  units and three units are assigned treatment, and four units are assigned to control (i.e., no treatment).

- ▶ Each unit's propensity score is ?.
- ▶ The probability of an particular treatment assignment is ?.

## The propensity score

```
i <- combn(1:8,3)
colnames(i) <- paste0(1:56, c("st Trt Assig", # rename columns
                             "nd Trt Assig",
                             "rd Trt Assig", rep("th Trt Assig", 53)))
row.names(i) <- paste0("Unit ", 1:3) # rename rows
i[,1:4] #print first four columns
```

	1st Trt Assig	2nd Trt Assig	3rd Trt Assig	4th Trt Assig
Unit 1	1	1	1	1
Unit 2	2	2	2	2
Unit 3	3	4	5	6

Each column corresponds to the units that will be treated; so the units that will not be treated are not included in the column. For example in the first treatment assignment units 1, 2, 3 will be treated and units 4, 5, 6, 7, 8 will be given control.

## The propensity score

- ▶ Consider a completely randomized design with  $n$  units and  $m$  units are assigned treatment.
- ▶ Each unit has probability  $\frac{m}{n}$  of receiving treatment (and  $1 - \frac{m}{n}$  of receiving control).
- ▶ Thus, each person's propensity score is  $m/n$ .
- ▶ The probability of a particular treatment assignment is  $\frac{1}{\binom{n}{m}}$ .