# STA305/1004 - Class 8

October 1, 2019

- Midterm test in one week
- TA office hours Thurs, Fri, Monday
- Thursday Class is a tutorial Class - Come with questions. or post questions on piazza.

# Today's Class

- Case study on power poses study: study replication and power
- ~~Sample size and power in studies with two proportions~~
- Calculating power via simulation
- Introduction to causal inference

# Can power poses significantly change outcomes in your life?


# Can power poses significantly change outcomes in your life?

Cuddy's study methods:

- Randomly assigned 42 participants to the high-power pose or the low-power-pose condition.
- Participants believed that the study was about the science of physiological recordings and was focused on how placement of electrocardiography electrodes above and below the heart could influence data collection.
- Participants' bodies were posed by an experimenter into high-power or low-power poses. Each participant held two poses for 1 min each.
- Participants' risk taking was measured with a gambling task; feelings of power were measured with self-reports.
- Saliva samples, which were used to test cortisol and testosterone levels, were taken before and approximately 17 min after the power-pose manipulation.

(Carney, Cuddy, Yap, 2010)

# Can power poses significantly change outcomes in your life?

Cuddy's study results:

> *As hypothesized, high-power poses caused an increase in testosterone compared with low-power poses, which caused a decrease in testosterone, $F(1, 39) = 4.29$, $p < .05$; $r = .34$. Also as hypothesized, high-power poses caused a decrease in cortisol compared with low-power poses, which caused an increase in cortisol, $F(1, 38) = 7.45$, $p < .02$; $r = .43$*

# Can power poses significantly change outcomes in your life?

- The study was replicated by Ranehill et al. (2015)
- An initial power analysis based on the effect sizes in Carney et al. (power = 0.8, $\alpha = .05$) indicated that a sample size of 100 participants would be suitable.

```
library(pwr)
pwr.t.test(d=0.6,power = 0.8)
```

```
     Two-sample t test power calculation

              n = 44.58577
              d = 0.6
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: n is number in *each* group
```

# Can power poses significantly change outcomes in your life?

- Ranehill et al. study used a sample of 200 participants to increase reliability.
- This study found none of the significant differences found in Cuddy's study.
- The replication study obtained very precise estimates of the effects.
- What happened?

# Can power poses significantly change outcomes in your life?

# Can power poses significantly change outcomes in your life?

- Sampling theory predicts that the variation between samples is proportional to $\frac{1}{\sqrt{n}}$.
- In small samples we can expect variability.
- Many researchers often expect that these samples will be more similar than sampling theory predicts.

# Study replication

Original study: $z = 2.23 = \dfrac{\bar{X}_{old} - 0}{\sigma/\sqrt{20}}$

New study: $H_0: \mu = 0$ vs $\mu > 0$. Calculate power at $\mu = \bar{X}_{old} = 2.23\dfrac{\sigma}{\sqrt{20}}$

New study rejects $\iff \dfrac{\bar{X}_{new}}{\sigma/\sqrt{10}} \geq 1.64$

Suppose that you have run an experiment on 20 subjects, and have obtained a significant result from a two-sided z-test ($H_0 : \mu = 0$ vs. $H_1 : \mu \neq 0$) which confirms your theory ($z = 2.23$, $p < 0.05$, two-tailed). The researcher is planning to run the same experiment on an additional 10 subjects. What is the probability that the results will be significant at the 5% level by a one-tailed test ($H_1 : \mu > 0$), seperately for this group?

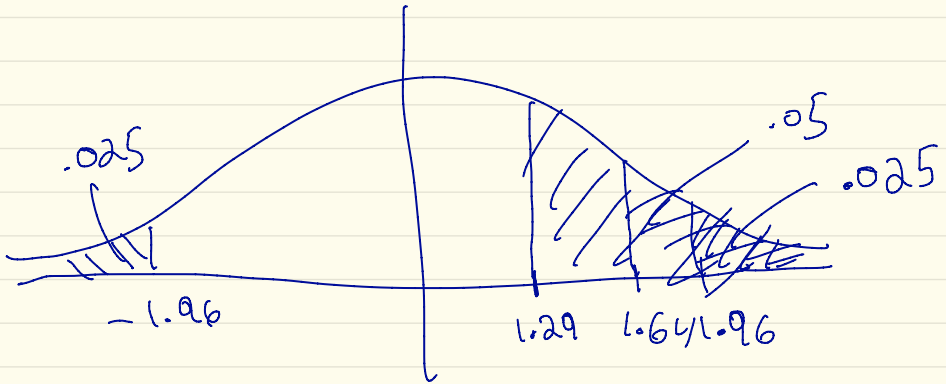
$$P\left(\bar{X}_{new} \geq \frac{\sigma}{\sqrt{10}} 1.64\right) = P\left(\frac{\bar{X}_{new} - 2.23\sigma}{\sigma/\sqrt{10}} \geq \frac{\frac{\sigma}{\sqrt{10}} 1.64 - 2.23\sigma}{\sigma/\sqrt{10}}\right)$$

($\bar{X}_{new} = \mu$ under the alternative; $1.29$)

$$= P\left(Z \geq 1.64 - 2.23\sqrt{\tfrac{10}{20}}\right)$$

$= \text{power} < 0.50$

A significant result in a small study has a low prob. of being reproducible.




$Z_{.025}$ 97.5 $^{th}$ percentile of Normal distribution.

# Calculating Power by Simulation

- If the test statistic and distribution of the test statistic are known then the power of the test can be calculated via simulation.
- Consider a two-sample t-test with 30 subjects per group and the standard deviation of the clinical outcome is known to be 1.
- What is the power of the test $H_0 : \mu_1 - \mu_2 = 0$ versus $H_a : \mu_1 - \mu_2 = 0.5$, at the 5% significance level?
- The power is the proportion of times that the test correctly rejects the null hypothesis in repeated sampling.

# Calculating Power by Simulation

generate random values from Normal distribution

We can simulate a single study using the `rnorm()` command. Let's assume that $n_1 = n_2 = 30, \mu_1 = 3.5, \mu_2 = 3, \sigma = 1, \alpha = 0.05$.

```
set.seed(2301)
t.test(rnorm(30,mean=3.5,sd=1),rnorm(30,mean=3,sd=1),var.equal = T)
```

```
    Two Sample t-test

data:  rnorm(30, mean = 3.5, sd = 1) and rnorm(30, mean = 3, sd = 1)
t = 2.1462, df = 58, p-value = 0.03605
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 0.03458122 0.99248595
sample estimates:
mean of x mean of y
 3.339362  2.825828
```

< .05

Should you reject $H_0$? Yes

# Calculating Power by Simulation

- Suppose that 10 studies are simulated.
- What proportion of these 10 studies will reject the null hypothesis at the 5% level?
- To investigate how many times the two-sample t-test will reject at the 5% level the `replicate()` command will be used to generate 10 studies and calculate the p-value in each study.
- It will still be assumed that $n_1 = n_2 = 30, \mu_1 = 3.5, \mu_2 = 3, \sigma = 1, \alpha = 0.05$.

```
set.seed(2301)
pvals <- replicate(10,t.test(rnorm(30,mean=3.5,sd=1),
                             rnorm(30,mean=3,sd=1),
                             var.equal = T)$p.value)
pvals # print out 10 p-values
```

```
 [1] 0.03604893 0.15477655 0.01777959 0.40851999 0.34580930 0.11131007
 [7] 0.14788381 0.00317709 0.09452230 0.39173723
```

```
#power is the number of times the test rejects at the 5% level
sum(pvals<=0.05)/10
```

```
[1] 0.3
```

# Calculating Power by Simulation

But, since we only simulated 10 studies the estimate of power will have a large standard error. So let's try simulating 10,000 studies so that we can obtain a more precise estimate of power.

```
set.seed(2301)
pvals <- replicate(10000,t.test(rnorm(30,mean=3.5,sd=1),
                                rnorm(30,mean=3,sd=1),
                               var.equal = T)$p.value)
sum(pvals<=0.05)/10000
```

```
[1] 0.4881
```

# Calculating Power by Simulation

This is much closer to the theoretical power obtained from `power.t.test()`.

```
power.t.test(n = 30,delta = 0.5,sd = 1,sig.level = 0.05)
```

```
     Two-sample t test power calculation

              n = 30
          delta = 0.5
             sd = 1
      sig.level = 0.05
          power = 0.477841
    alternative = two.sided

NOTE: n is number in *each* group
```

# Calculating Power by Simulation

- The built-in R functions `power.t.test()` and `power.prop.test()` don't have an option for calculating power where the there is unequal allocation of subjects between groups.
- These built-in functions don't have an option to investigate power if other assumptions don't hold (e.g., normality).
- One option is to simulate power for the scenarios that are of interest. Another option is to write your own function using the formula derived above.

# Calculating Power by Simulation

- Suppose the standard treatment for a disease has a response rate of 20%, and an experimental treatment is anticipated to have a response rate of 28%.
- The researchers want both arms to have an equal number of subjects.
- A power calculation above revealed that the study will require $446 \times 2 = 892$ patients for 80% power.
- What would happen to the power if the researchers put more patients in the experimental arm compared to the control arm?

# Calculating Power by Simulation

Bin(1500, .28)

Patient 1 2 3 ... 1500

1 0 1 0 1 0

- The number of subjects in the experimental arm that have a positive response to treatment will be an observation from a $Bin(1500, 0.28)$.
- The number of subjects that have a positive response to the standard treatment will be an observation from a $Bin(500, 0.2)$.
- We can obtain simulated responses from these distributions using the `rbinom()` command in R.

```
set.seed(2301)
rbinom(1,1500,0.28)
```

```
[1] 403
```

403 subjects had positive response

1500 − 403 subjects had neg. response.

```
rbinom(1,500,0.20)
```

```
[1] 89
```

89 subjects had a pos. response.

# Calculating Power by Simulation

- The p-value for this simulated study can be obtained using `prop.test()`.

```
set.seed(2301)
prop.test(x=c(rbinom(1,1500,0.28),rbinom(1,500,0.20)),
          n=c(1500,500),correct = F)
```

$X_1 = 403$ $X_2 = 89$

$n_1$ $n_2$

Continuity Correction for test of Proportions.

```
    2-sample test for equality of proportions without continuity
    correction

data:  c(rbinom(1, 1500, 0.28), rbinom(1, 500, 0.2)) out of c(1500, 500
X-squared = 16.62, df = 1, p-value = 4.568e-05
alternative hypothesis: two.sided
95 percent confidence interval:
 0.05032654 0.13100680
sample estimates:
   prop 1    prop 2
0.2686667 0.1780000
```

$< .05$

$\alpha$ reject $H_0$.

# Calculating Power by Simulation

Assume the standard treatment for a disease has a response rate of 20%, and an experimental treatment is anticipated to have a response rate of 28%.

```
set.seed(2301)
n1 <- 300
n2 <- 100
pvals <- replicate(10000,
prop.test(x=c(rbinom(n = 1,size = n1,prob = 0.28),
              rbinom(n = 1,size = n2,prob = 0.20)),
          n=c(n1, n2),correct = F)$p.value)
sum(pvals <= 0.01)
```

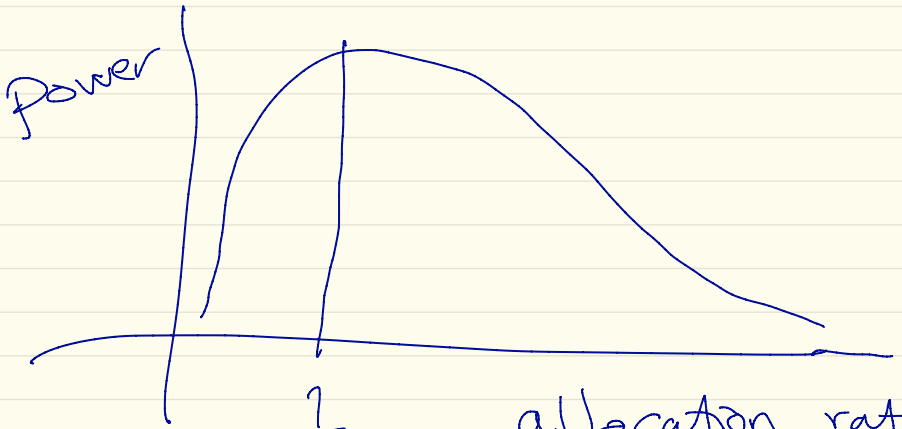
Suppose $n_1 = 500$ instead of 300 - what would happen to power?

```
## [1] 1434
```

If the researchers enrol 300 subjects in the experimental arm, and 100 subjects in the standard arm then the power is 0.1434, at the 0.01 significance level. Power was calculated by simualting 10000 hypothetical studies.

1434 / 10000

If $\alpha = 0.05$ then power $> 0.1434$




↑ equal number in each group.

look back at previous slides.

# Question

Respond at **PollEv.com/nathantaback**
Text **NATHANTABACK** to **37607** once to join, then **1, 2, 3, 4, or 5**

**If the researchers enrol A subjects in the experimental arm, and B subjects in the standard arm then the power is C, at the D significance level. Power was calculated by simulating E hypothetical studies. The values A, B, C, D, E are:**

| | |
|---|---|
| A = 100, B = 300, C = 0.1434, D = 0.01, E = 400 | **1** |
| A = 300, B = 100, C = 1434, D = 0.01, E = 10000 | **2** |
| A = 300, B = 100, C = 0.1434, D = 0.01, E = 10000 | **3** |
| A = 300, B = 100, C = 0.1434, D = 0.05, E = 10000 | **4** |
| A = 100, B = 300, C = 0.1434, D = 0.05, E = 10000 | **5** |

3%
80%
17%

# Introduction to causal inference - Bob's headache

- Suppose Bob, at a particular point in time, is contemplating whether or not to take an aspirin for a headache.
- There are two treatment levels, taking an aspirin, and not taking an aspirin.
- If Bob takes the aspirin, his headache may be gone, or it may remain, say, an hour later; we denote this outcome, which can be either "Headache" or "No Headache," by $Y(\text{Aspirin})$.
- Similarly, if Bob does not take the aspirin, his headache may remain an hour later, or it may not; we denote this potential outcome by $Y(\text{No Aspirin})$, which also can be either "Headache," or "No Headache."
- There are therefore two potential outcomes, $Y(\text{Aspirin})$ and $Y(\text{No Aspirin})$, one for each level of the treatment. The causal effect of the treatment involves the comparison of these two potential outcomes.

# Introduction to causal inference - Bob's headache

Because in this example each potential outcome can take on only two values, the unit- level causal effect – the comparison of these two outcomes for the same unit – involves one of four (two by two) possibilities:

1. Headache gone only with aspirin: Y(Aspirin) = No Headache, Y(No Aspirin) = Headache
2. No effect of aspirin, with a headache in both cases: Y(Aspirin) = Headache, Y(No Aspirin) = Headache
3. No effect of aspirin, with the headache gone in both cases: Y(Aspirin) = No Headache, Y(No Aspirin) = No Headache
4. Headache gone only without aspirin: Y(Aspirin) = Headache, Y(No Aspirin) = No Headache

# Introduction to causal inference - Bob's headache

There are two important aspects of this definition of a causal effect.

1. The definition of the causal effect depends on the potential outcomes, but it does not depend on which outcome is actually observed.
2. The causal effect is the comparison of potential outcomes, for the same unit, at the same moment in time post-treatment.

- The causal effect is not defined in terms of comparisons of outcomes at different times, as in a before-and-after comparison of my headache before and after deciding to take or not to take the aspirin.

# The fundemental problem of causal inference

"The fundamental problem of causal inference" (Holland, 1986, p. 947) is the problem that at most one of the potential outcomes can be realized and thus observed.

- If the action you take is Aspirin, you observe $Y(\text{Aspirin})$ and will never know the value of $Y(\text{No Aspirin})$ because you cannot go back in time.
- Similarly, if your action is No Aspirin, you observe $Y(\text{No Aspirin})$ but cannot know the value of $Y(\text{Aspirin})$.
- In general, therefore, even though the unit-level causal effect (the comparison of the two potential outcomes) may be well defined, by definition we cannot learn its value from just the single realized potential outcome.

# The fundemental problem of causal inference

The outcomes that would be observed under control and treatment conditions are often called **counterfactuals** or **potential outcomes**.

- If Bob took asprin for his headache then he would be assigned to the treatment condition so $T_i = 1$.
- Then $Y(\text{Aspirin})$ is observed and $Y(\text{No Aspirin})$ is the unobserved counterfactual outcome—it represents what would have happened to Bob if he had no taken aspirin.
- Conversely, if Bob had not taken aspirin then $Y(\text{No Aspirin})$ is observed and $Y(\text{Aspirin})$ is counterfactual.
- In either case, a simple treatment effect for Bob can be defined as

$$\text{treatment effect for Bob} = Y(\text{Aspirin}) - Y(\text{No Aspirin}).$$