STA305/1004-Class 20

Dec. 3, 2019

Today's Class

Fractional factorial design

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- Full factorials are seldom used in practice for large k (k>=7).
- For economic reasons fractional factorial designs, which consist of a fraction of full factorial designs are used.



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- Polymer solutions were prepared and spread as a film on a microscope slide. Six different responses were recorded.
- Five factors (each with 2 levels) were studied in an 8 run 2^k factorial experiment.
- How many runs are usually required to study five factors, each with 2 levels, in a factorial experiment? $32 = 2^{5}$

run	А	В	С	D	Е	y1	y2	у3	y4	y5	уб
1	-1	-1	-1	1	-1	no	no	yes	no	slightly	yes
2	1	-1	-1	1	1	no	yes	yes	yes	slightly	yes
3	-1	1	-1	-1	1	no	no	no	yes	no	no
4	1	1	-1	-1	-1	no	yes	no	no	no	no
5	-1	-1	1	-1	1	yes	no	no	yes	no	slightly
6	1	-1	1	-1	-1	yes	yes	no	no	no	no
7	-1	1	1	1	-1	yes	no	yes	no	slightly	yes
8	1	1	1	1	1	yes	yes	yes	yes	slightly	yes

Factors	-	+
A: catalyst (%)	1	1.5
B: additive (%)	1/4	1/2
C: emulsifier P (%)	2	3
D: emulsifier Q (%)	1	2
E: emulsifier R (%)	1	2

y1 - Hazy?, y2 - Adheres?, y3 - Grease on Top of Film?, y4 - Grease Under Film?, y5 - Dull, Adjusted pH, y6 - Dull original pH

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- ► The column of signs associated with the BC interaction was used to accommodate factor D, the ABC interaction column was used for factor E.

ABC	BC=D	A B C = E
1-1-1	+1	-1
	+1	+1
-1 +1 -1	-1	+1
+1+1-1	- 1	- I + I
-1-1+1	- 1	- 1
41-141	- (-1
+1+1+1	+ (+ (+(

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- ▶ Only 1/4 were run. This design is called a quarter fraction of the full 2⁵ or a 2⁵⁻² design (a two to the five minus two design).
- ► In general a 2^{k-p} design is a $\frac{1}{2^p}$ fraction of a 2^k design using 2^{k-p} runs. This design can study k factors in $\frac{1}{2^p}$ fraction of the runs. k = 5, p = 2

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- A chemist in an industrial development lab was trying to formulate a household liquid product using a new process.
- The liquid had good properties but was unstable.
- The chemist wanted to synthesize the product in hope of hitting conditions that would give stability, but without success.
- The chemist identified four important influences: A (acid concentration), B (catalyst concentration), C (temperature), D (monomer concentration).

				1	- HV
test	А	В	С	D	у
1	-1	-1	-1	-1	20
2	1	-1	-1	1	14
3	-1	1	-1	1	17
4	1	1	-1	-1	10
5	-1	-1	1	1	19
6	1	-1	1	-1	13
7	-1	1	1	-1	14
8	1	1	1	1	10

His 8 run fractional factorial design is shown below.

▶ The signs of the ABC interaction is used to accommodate factor D. The tests were run in random order. He wanted to achieve a stability value of at least 25.

Missing : D=ABC fact.prod <- lm(y~A*B*C*D,data=tab0602)</pre> fact.prod1 <- aov(y~A*B*C*D,data=tab0602)</pre> round(2*fact.prod\$coefficients,2) (Intercept) Α В С A:B 0.75 29.25 -5.75-3.75-1.25A:C B:C A:D B:D :B:(0.75 -0.25NA NA NA A:B:D A:C:D B:C:D A:B:C:D NΑ NA NA NΑ

Even though the stability never reached the desired level of 25, two important factors, A and B, were identified.



Poll Question



N=24

Factors	Number of effects
Main	5
2-factor	10
3-factor	10
4-factor	5
5-factor	1

What information could have been obtained if a full 2^5 design had been used?

▶ 31 degrees of freedom in a 32 run design.

What information could have been obtained if a full 2⁵ design had been used?

Factors	Number of effects		
Main	5		
2-factor	10		
3-factor	10	10 -	
4-factor	5	160	
5-factor	1		

► 31 degrees of freedom in a 32 run design.

16 used for estimating three factor interactions or higher.

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2-factor	10
3-factor	10
4-factor	5
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What information could have been obtained if a full 2⁵ design had been used?

- ▶ 31 degrees of freedom in a 32 run design.
- ▶ 16 used for estimating three factor interactions or higher.
- Is it practical to commit half the degrees of freedom to estimate such effects?

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What information could have been obtained if a full 2⁵ design had been used?

- ▶ 31 degrees of freedom in a 32 run design.
- ▶ 16 used for estimating three factor interactions or higher.
- Is it practical to commit half the degrees of freedom to estimate such effects?
- According to effect hierarchy principle three-factor and higher not usually important.
- Thus, using full factorial wasteful. It's more economical to use a fraction of full factorial design that allows additional lower order effects to be estimated.

Consider a design that studies five factors in 16 run. A half fraction of a 2^5 or 2^{5-1} .

				-		E=BCD
Run	В	С	D	Е	Q	
1	-1	1	1	-1	-1	
2	1	1	1	1	-1	
3	-1	-1	1	1	-1	
4	1	-1	1	-1	-1	
5	-1	1	-1	1	-1	
6	1	1	-1	-1	-1	
7	-1	-1	-1	-1	-1	
8	1	-1	-1	1	-1	
9	-1	1	1	-1	1	
10	1	1	1	1	1	
11	-1	-1	1	1	1	
12	1	-1	1	-1	1	
13	-1	1	-1	1	1	
14	1	1	-1	-1	1	
15	-1	-1	-1	-1	1	
16	1	-1	-1	1	1	

- ▶ The factor E is assigned to the column BCD.
- ▶ The column for E is used to estimate the main effect of E and also for BCD.
- ▶ The main factor E is said to be **aliased** with the BCD interaction.

E = B C D $\Rightarrow E \cdot E = B C D E$ This aliasing relation is denoted by I = S C D EE = B C D or I = B C D E,

where I denotes the column of all +'s.

- This uses same mathematical definition as the confounding of a block effect with a factorial effect.
- Aliasing of the effects is the trade-off one must make for choosing a smaller design.
- ▶ The 2⁵⁻¹ design has only 15 degrees of freedom for estimating factorial effects, it cannot estimate all 31 factorial effects among the factors B, C, D, E, Q.

- The equation I = BCDE is called the **defining relation** of the 2^{5-1} design.
- The design is said to have resolution IV because the defining relation consists of the "word" BCDE, which has "length" 4.
- Multiplying both sides of I = BCDE by column B

$$B = B \times I = B \times BCDE = CDE$$
,

the relation B = CDE is obtained.

B is aliased with the CDE interaction. Following the same method all <u>15 aliasing</u> relations can be obtained.

$$E=BCP, B=CDE, C=BDE, D=BCE$$

$$I=BCDE=E$$

$$Q=BCDEQ, QE=ABCD, QE=CDE, QE=CE, QE$$

- ► To get the most desirable alias patterns, fractional factorial designs of highest resolution would usually be employed.
- > There are important exceptions to this rule that we will not cover in the course.

Class Question

Consider a 2^{5-1} fractional factorial design.

- (a) How many factors does this design have? 5 factors (b) How many runs are involved in this design? $l_6 = 2^{s-1} = 2^{s}$
- (c) How many levels for each factor?
- (d) The factor E is assigned to the four-way interaction (ABCD). What is the defining relation? What is the design resolution? What are the aliasing relations?

An experiment to improve a heat treatment process on truck leaf springs (Wu and Hamada (2009)). The height of the unloaded spring (y) is an important quality characteristic.



Five factors that might affect height (y) were studied in this 2^{5-1} design.

Factor	Level
B. Temperature	1840 (-), 1880 (+)
C. Heating time	23 (-), 25 (+)
D. Transfer time	10 (-), 12 (+)
E. Hold down time	2 (-), 3 (+)
Q. Quench oil temperature	130-150 (-), 150-170 (+)

The factor E is assigned the column of the three-way interaction between B, C, and D.

В	С	D	Е	Q	у
-1	1	1	-1	-1	7.79
1	1	1	1	-1	8.07
-1	-1	1	1	-1	7.52
1	-1	1	-1	-1	7.63
-1	1	-1	1	-1	7.94
1	1	-1	-1	-1	7.95
-1	-1	-1	-1	-1	7.54
1	-1	-1	1	-1	7.69
-1	1	1	-1	1	7.29
1	1	1	1	1	7.73
-1	-1	1	1	1	7.52
1	-1	1	-1	1	7.65
-1	1	-1	1	1	7.40
1	1	-1	-1	1	7.62
-1	-1	-1	-1	1	7.20
1	-1	-1	1	1	7.63

I=BCOE

Questions: (1) What is the defining relation? (2) What is the design resolution? (3) What are the aliasing relations?

See previous Stide.

T

Example - Leaf	spring experi	ment	<u> </u>	BCDE		
Note: this	are factoria	Since mult.	by BE	= CD	5 Floot of	¢
The factorial e	ffects are estimat	ted as before.	[actorial	Eta	uu.
fact.leaf <-	lm(y~B*C*D*E*	Q,data=leafs	pring)	. 2+	0.2	
round(2*fact	.leaf\$coeffici	ents,2)	Y	, (> -		
(Intercept)	В	С	D	Е	Q	
15.27	0.22	0.18	0.03	0.10	-0.26	
B:C	B:D	C:D	B:E	C:E	D:E	
0.02	0.02	-0.04	NA	NA	NA	
B:Q	C:Q	D:Q	E:Q	B:C:D	B:C:E	
0.08	(-0.17	0.05	0.03	NA	NA	
B:D:E	C:D:E	B:C:Q	B:D:Q	C:D:Q	B:E:Q	
NA	NA	0.01	-0.04	-0.05	NA	
C:E:Q	D:E:Q	B:C:D:E	B:C:D:Q	B:C:E:Q	B:D:E:Q	
NA	NA	NA	NA	NA	NA	
C:D:E:Q	B:C:D:E:Q					
NA	NA					

Questions: (1) Why are some effects NA? (2) What is the factorial effect of B? Aliasing produced NA For Some effects. C-9-5 BE and CD are aliasco. This is one of the effects of using I= BCDE as defining relation



Question: Interpret the interaction between Heating Time (C) and Quench oil temperature (Q).



Interpretation of interaction between Cand Q: The average change in Quench Oil temp When heating time is high vs. low. Mean Change C= +1

Mean Change C=-1



B:Q

factors

D:Q

B:C:Q

B:D:Q C:D:Q



B:C B:D

D E